Enrich, XII(I): 16 - 20, July – Dec., 2020

2 −Total Difference Cordial Labeling for Some Zero-Divisor Graphs

[1]K. Aruna Sakthi, [2]R. Rajeswari, [3]N. Meenakumari
[1] Research Scholar Reg: No:20212012092006 , [2] Assistant Professor [3]Associate Professor
PG & Research Department of Mathematics
A.P.C. Mahalaxmi College for women, Thoothukudi

Abstract: Let $G = (V, E)$ be a graph. A function $f: V(G) \to \{0,1,2, \dots, k-1\}$ be a map where $k \in \mathbb{N}$ and $k > 1$. For each edge uv assign the label $|f(u) - f(v)|$. f is called k-total difference cordial labeling of G if $\left|t_{df}(i) - t_{df}(0)\right| \leq 1$, $i, j \in \{0,1,2, \dots, k-1\}$ where $t_{df}(x)$ denotes the total number of vertices and labelled with x. A graph which admits k-total difference cordial labeling is called k-total difference cordial graphs. In this paper we probe into some of the zero-divisor graph for the 2-total difference cordial labeling.

Keywords: Zero-divisor graphs, 2-Total difference cordial labeling.
AMS Classification: 05C78, 05C25

1. Introduction

There are two variations of the zero-divisor graph. One is in the Beck definition in the year 1988, in which the vertices represent all elements of the ring [2]. In the year 1999, Anderson and Livingston slightly varied the graph, in which the vertices represent only the zero-divisor of the given ring [1]. Graph labeling was introduced by Alexander Rosa in the year 1967[4,11]. Rosa identified three types of labeling which was later renamed by Solomon Golomb[4]. Further developed by Grahamm and Slone in 1980[5]. In the field of Engineering and technology labelled graphs has its own application. Cordial labeling was introduced by Cahit [3] as a weaker version of graceful and harmonious labeling of graphs, which was later extended by Hovey [6]. In [7], Ponraj et al. introduced a new labeling called difference cordial labeling. In [7 8 9 10 11], difference cordial labeling behavior of several graphs like path, cycle, complete graph, complete bipartite graph, bistar, wheel, web and some more standard graphs have been investigated. Seoud and Salman [13], studied the difference cordial labeling behaviour of some families of graphs and they are ladder, triangular ladder, grid, step ladder and two sided step ladder graphs etc. In this paper we have investigated for the some of the zero-divisor graphs.

2.Preliminaries

Definition 2.1 Zero-Divisor Graph

Let R be a commutative ring with identity 1 and let $Z(R)$ be its set of zero-divisors. We associate a $\Gamma(R)$ to R with vertices $Z^* = Z(R) - \{0\}$, the set nonzero zero-divisor of R, and for distinct $x, y \in Z(R)^*$, the vertices x and y are adjacent if and only if $xy = 0$. We denote their zero-divisor graph of R by $\Gamma_0(R)$ if we take vertex set as Z (R). In $\Gamma_0(R)$, the vertex 0 is adjacent to every other vertex. $\Gamma(R)$ is a induced subgraph of $\Gamma_0(R)$.

Definition 2.2 2-Total difference cordial labeling

Let $G = (V, E)$ be a graph. A function $f: V(G) \to \{0,1,2, \dots, k-1\}$ be a map where $k \in \mathbb{N}$ and $k > 1$. For each edge uv assign the label $|f(u) - f(v)|$. f is called k-total difference cordial labeling of G if $\left|t_{df}(i) - t_{df}(0)\right| \leq 1$, $i, j \in \{0,1,2, \dots, k-1\}$ where $t_{df}(x)$ denotes the total

number of vertices and labelled with x. A graph which admits k-total difference cordial labeling is called k-total difference cordial graphs.
In this paper results has been proved for 2-Total difference cordial labeling for some zero-divisor graphs.

3. 2-Total difference cordial labeling

Theorem: 3.1 The zero-divisor graph $\Gamma(Z_{2p})$, $p \geq 3$ be a prime number admits 2-total difference cordial labeling.

Proof: Let graph $G = \Gamma(Z_{2p})$ and $p \geq 3$ be a prime number.

The vertex set of G is $V(G) = \{x_1, x_2, \dots, x_{p-1}, x_p\} = \{2,4,\dots,2(p-1), p\}$

The edge set of G is $E(G) = \{x_i x_p / 1 \leq i \leq p-1\}$

$|V(G)| = p$; $|E(G)| = p-1$.

Define a function $f: V(G) \to \{0,1\}$ such that $f(x_i) = 0$ $1 \leq i \leq p-1$ and $f(x_p) = 1$ and the edge $x_i x_p$ assigns the label $f(x_i x_p) = |f(x_i) - f(x_p)|$; $1 \leq i \leq p-1$.

The total difference of 0 $t_{df}(0) = p-1$.

The total difference of 1 $t_{df}(1) = p$.

Therefore $|t_{df}(1) - t_{df}(0)| = |p - p + 1| = 1$, since it satisfies the condition.

Therefore the graph G admits 2-total difference cordial labeling.

Theorem: 3.2 The zero-divisor graph $\Gamma(Z_{3p})$, $p > 3$ be a prime number admits 2-total difference cordial labeling.

Proof: Let graph G= $\Gamma(Z_{3p})$ and $p > 3$, be a prime number.

The vertex set of G is $V(G) = \{t_1, t_2, x_1, x_2, x_3, \dots, x_{p-1}\} = \{p, 2p, 3,6,9, \dots, 3(p-1)\}$

The edge set of G is $E(G) = \{t_i x_j / 1 \leq i \leq 2, 1 \leq j \leq p-1\}$

$|V(G)| = p+1$; $|E(G)| = 2p-2$.

Define $f: V(G) \to \{0,1\}$ such that

$$f(t_i) = \begin{cases} 1 \; if \; i \cong 1 mod 2 \\ 0 \; if \; i \cong 2 mod 2 \end{cases}; i = 1,2 \text{ and } f(x_j) = \begin{cases} 0 \; if \; i \cong 1 mod 2 \\ 1 \; if \; i \cong 0 mod 2 \end{cases}; \; 1 \leq j \leq p-1$$

and the edge $t_i x_j$ assigns the label $f(t_i x_j) = |f(t_i) - f(x_j)|$; $1 \leq i \leq 2$ and $1 \leq j \leq p-1$.

The total difference $t_{df}(0) = \frac{3p-1}{2}$.

The total difference of 1 $t_{df}(1) = \frac{3p-1}{2}$.

Therefore $|t_{df}(1) - t_{df}(0)| = \left|\frac{3p-1}{2} - \frac{3p-1}{2}\right| = 0$, since it satisfies the condition.

Therefore the graph G admits 2-total difference cordial labeling.

Theorem: 3.3 The zero-divisor graph $\Gamma(Z_{5p})$, $p \geq 3$ be a prime number admits 2-total difference cordial labeling.

Proof: Let graph $G = \Gamma(Z_{5p})$, $p \geq 3$ be a prime number.

The vertex set of G is $V(G) = \{t_1, t_2, t_3, t_4, x_1, x_2, x_3, \dots, x_{p-1}\}$
$= \{p, 2p, 3p, 4p, 5,10,15, \dots, 5(p-1)\}$

The edge set of G is $E(G) = \{t_i x_j / 1 \leq i \leq 4, 1 \leq j \leq p-1\}$

$|V(G)| = p+3$; $|E(G)| = 4p-4$.

Define $f: V(G) \to \{0,1\}$ such that

$f(t_i) = \begin{cases} 1 \; if \; i \cong 1mod2 \\ 0 \; if \; i \cong 2mod2 \end{cases}; 1 \leq i \leq 4$ and $f(x_j) = \begin{cases} 0 \; if \; i \cong 1mod2 \\ 1 \; if \; i \cong 0mod2 \end{cases}; \; 1 \leq j \leq p-1$

and the edge $t_i x_j$ assigns the label $f(t_i x_j) = |f(t_i) - f(x_j)|; 1 \leq i \leq 4$ and $1 \leq j \leq p-1$.

The total difference $t_{df}(0) = \frac{5p-1}{2}$.

The total difference of 1 $t_{df}(1) = \frac{5p-1}{2}$.

Therefore $|t_{df}(1) - t_{df}(0)| = \left|\frac{5p-1}{2} - \frac{5p-1}{2}\right| = 0,$ since it satisfies the condition.

Therefore the graph G admits 2-total difference cordial labeling.

Theorem: 3.4 For the prime number $p, q \geq 3$ and $p < q$ and $p \neq q$ the zero-divisor graph $\boldsymbol{\Gamma(Z_{pq})}$ admits 2- total difference cordial labeling.

Proof: Let graph $G = \Gamma(Z_{pq})$, $p, q \geq 3$ and $p < q$ and $p \neq q$ where p, q are prime numbers.

The vertex set of G can be partitioned in to two sets $V_1(G)$ and $V_2(G)$

$V_1(G) = \{x_1, x_2, x_3, \dots, x_{q-1}\} = \{p, 2p, \dots, (q-1)p\}$

$V_2(G) = \{t_1, t_2, t_3, \dots, t_{p-1}\} = \{q, 2q, 3q, \dots, (p-1)q\}$

The edge set of G is $E(G) = \{x_i t_j / 1 \leq i \leq q-1 \; and \; 1 \leq j \leq p-1\}$

$|V(G)| = p+3$; $|E(G)| = 4p-4$.

Define $f: V(G) \to \{0,1\}$ such that

$f(x_i) = \begin{cases} 1 \; if \; i \cong 1mod2 \\ 0 \; if \; i \cong 2mod2 \end{cases}; 1 \leq i \leq q-1$ and $f(t_j) = \begin{cases} 0 \; if \; i \cong 1mod2 \\ 1 \; if \; i \cong 0mod2 \end{cases}; \; 1 \leq j \leq p-1$

And the edge $x_i t_j$ assigns the label $f(x_i t_j) = |f(x_i) - f(t_j)|; 1 \leq i \leq q-1$ and $1 \leq j \leq p-1$.

The total difference $t_{df}(0) = \frac{pq-1}{2}$.

The total difference of 1 $t_{df}(1) = \frac{pq-1}{2}$.

Therefore $|t_{df}(1) - t_{df}(0)| = \left|\frac{pq-1}{2} - \frac{pq-1}{2}\right| = 0,$ since it satisfies the condition.

Therefore the graph G admits 2-total difference cordial labeling.

Theorem: 3.5 For any prime number $p \geq 3$, the zero-divisor graph $\boldsymbol{\Gamma(Z_2 \times Z_p)}$ admits 2-total difference cordial labeling.

Proof: Let graph $G = \Gamma(Z_2 \times Z_p)$, $p \geq 3$ be a prime number.

The vertex set of G is $V(G) = \{x_1, x_2, x_3, \dots, x_{p-1}, t\}$

$= \{(0,1), (0,2), (0,3), \dots, (0, p-1), (1,0)\}$

The edge set of G is $E(G) = \{x_i t / 1 \leq i \leq p-1\}$

$|V(G)| = p$; $|E(G)| = p-1$.

Define $f: V(G) \to \{0,1\}$ such that $(x_i) = \begin{cases} 1 \; if \; i \cong 0mod2 \\ 0 \; if \; i \cong 1mod2 \end{cases}; 1 \leq i \leq p-1$ and $f(t) = 0$ and the edge $x_i t$ assigns the label $f(x_i t) = |f(x_i) - f(t)|; 1 \leq i \leq p-1$.

The total difference $t_{df}(0) = p$.

The total difference of 1 $t_{df}(1) = p-1$.

Therefore $|t_{df}(1) - t_{df}(0)| = |p-1-p| = 1,$ since it satisfies the condition.

Therefore the graph G admits 2-total difference cordial labeling.

Theorem: 3.6 The zero-divisor graphs $\boldsymbol{\Gamma(Z_{2p}) + \Gamma(Z_4)}$, $p \geq 3$ be a prime number admits 2-total difference cordial labeling.

Proof: Let graph $G = \Gamma(Z_{2p}) + \Gamma(Z_4)$, $p \geq 3$ be a prime number.

The vertex set of G is $V(G) = \{x_1, x_2, x_3, \dots, x_{p-1}, x_p, t\}$ = $\{2,4,6, \dots, 2(p-1), p, 2\}$ where $2 \in Z_4$.

The edge set of G is $E(G) = \{x_i t,\ x_p x_i,\ x_p t / 1 \leq i \leq p-1\}$.

$|V(G)| = p + 1$; $|E(G)| = 2p - 1$.

Define $f: V(G) \to \{0,1\}$ such that

$f(t) = 1;\ f(x_i) = \begin{cases} 1\ if\ i \cong 0 mod 2 \\ 0\ if\ i \cong 1 mod 2 \end{cases}$; $1 \leq i \leq p-1$ and $f(x_p) = 0$,

and the edge $x_i t, x_p x_i,\ x_p t$ assigns the label $f(x_i t) = |f(x_i) - f(t)|$;

$f(x_i x_p) = |f(x_i) - f(x_p)|$; $1 \leq i \leq p-1$. $f(x_p t) = |f(x_p) - f(t)|$;

The total difference $t_{df}(0) = \frac{3p-1}{2}$.

The total difference of 1 $t_{df}(1) = \frac{3p+1}{2}$.

Therefore $|t_{df}(1) - t_{df}(0)| = \left|\frac{3p+1}{2} - (\frac{3p-1}{2})\right| = 1$, since it satisfies the condition.

Therefore the graph G admits 2-total difference cordial labeling.

Theorem: 3.7 The zero-divisor graphs $\boldsymbol{\Gamma(Z_{2p}) + \Gamma(Z_6)}$, $p \geq 3$ be a prime number admits 2-total difference cordial labeling.

Proof: Let graph $G = \Gamma(Z_{2p}) + \Gamma(Z_6)$, $p \geq 3$ be a prime number.

The vertex set of G is $V(G) = \{x_1, x_2, x_3, \dots, x_{p-1}, x_p, s, t, u\}$ = $\{2,4,6, \dots, 2(p-1), p, 2,3,4\}$ where $2,3,4 \in Z_6$.

The edge set of G is $E(G) = \{x_i s,\ x_i t,\ x_i u,\ x_p x_i,\ x_p s, x_p t,\ x_p u, st, tu / 1 \leq i \leq p-1\}$.

$|V(G)| = p + 3$; $|E(G)| = 4p + 1$.

Define $f: V(G) \to \{0,1\}$ such that $f(s) = 0, f(t) = 0, f(u) = 1$,

$f(x_i) = \begin{cases} 1\ if\ i \cong 1 mod 2 \\ 0\ if\ i \cong 0 mod 2 \end{cases}$; $1 \leq i \leq p-1$ and $f(x_p) = 1$.

and the edge $x_i s,\ x_i t,\ x_i u,\ x_p x_i,\ x_p s, x_p t,\ x_p u, st, tu$ assigns the label

$f(x_i t) = |f(x_i) - f(t)|$; $f(x_i s) = |f(x_i) - f(s)|$; $f(x_i u) = |f(x_i) - f(u)|$

$f(x_i x_p) = |f(x_i) - f(x_p)|$; $1 \leq i \leq p-1$; $f(x_p t) = |f(x_p) - f(t)|$;

$f(x_p s) = |f(x_p) - f(s)|$; $f(x_p u) = |f(x_p) - f(u)|$; $f(st) = |f(s) - f(t)|$;

$f(tu) = |f(t) - f(u)|$.

The total difference $t_{df}(0) = 2p + 1$.

The total difference of 1 $t_{df}(1) = 2p + 2$.

Therefore $|t_{df}(1) - t_{df}(0)| = |2p + 2 - (2p + 1)| = 1$, since it satisfies the condition.

Therefore the graph G admits 2-total difference cordial labeling.

Theorem: 3.8 The zero-divisor graphs $\boldsymbol{\Gamma(Z_{2p}) + \Gamma(Z_9)}$, $p \geq 3$ be a prime number admits 2-total difference cordial labeling.

Proof: Let graph $G = \Gamma(Z_{2p}) + \Gamma(Z_9)$, $p \geq 3$ be a prime number.

The vertex set of G is $V(G) = \{x_1, x_2, x_3, \dots, x_{p-1}, x_p, s, t\}$ = $\{2,4,6, \dots, 2(p-1), p, 3,6\}$ where $3,6 \in Z_9$.

The edge set of G is $E(G) = \{x_i s, x_i t\ \ x_p x_i,\ x_p s, x_p t, st/1 \leq i \leq p-1\}$.
$|V(G)| = p+2$; $|E(G)| = 3p$.
Define $f: V(G) \rightarrow \{0,1\}$ such that $f(s) = 1, f(t) = 0$,
$f(x_i) = \begin{cases} 1\ if\ i \cong 1 mod 2 \\ 0\ if\ i \cong 0 mod 2 \end{cases}$; $1 \leq i \leq p-1$ and $f(x_p) = 1$
and the edge $x_i s, x_i t\ \ x_p x_i,\ x_p s, x_p t, st$ assigns the label
; $f(x_i s) = |f(x_i) - f(s)|$; $f(x_i t) = |f(x_i) - f(t)|$;
$f(x_i x_p) = |f(x_i) - f(x_p)|$; $1 \leq i \leq p-1$; $f(x_p s) = |f(x_p) - f(s)|$;
$f(x_p t) = |f(x_p) - f(t)|$; $f(st) = |f(s) - f(t)|$.
The total difference $t_{df}(0) = \frac{3p+1}{2}$.
The total difference of 1 $t_{df}(1) = \frac{3p+3}{2}$.
Therefore $|t_{df}(1) - t_{df}(0)| = \left|\frac{3p+3}{2} - (\frac{3p+1}{2})\right| = 1$, since it satisfies the condition.
Therefore the graph G admits 2-total difference cordial labeling.

References

[1] Anderson, David F. Livingston, Philip S. (1999). ***"The zero-divisor graph of a commutative ring"***, Journal of Algebra, 217(2): 434-447, doi:10.1006/jabr. 1998.7840 MR 1700509.
[2] Beck, I Stvan (1988), ***"Coloring of commutative rings"***. Journals of Algebra 116(1): 208-226 doi:10.1016/0021-8693(88)90202-5 MR 0944156.
[3] I.Cahit, "***Cordial graphs: a weaker version of graceful and harmonious graphs***", Ars Combin. 23(1987)201-207.

[4] Gallian, Joseph. A, ***"A dynamic survey of Graph labelings"*** 1996-2005. The Electronic Journal of Combinatorics.
[5] Graham. R.L and N.J.A. Sloane, ***"On additive bases and harmonious graphs"***, SIAM J. Alg. Disc, Meth, I, 382-404, 1980.
[6] M. Hovey, ***"A-Cordial graphs"***, Discrete Math. 93(1991) 183-194.

[7] R. Ponraj, S. Sathish Narayanan and R. Kala, "***Difference cordial labeling of graphs***", Global Journal of Mathematical Sicences: Theory and Practical, **5**(2013), 185-196.

[8] R. Ponraj, S. Sathish Narayanan and R.Kala, "***Difference cordial labeling of corona graphs"***, J.Math. Comput. Sci., **3**(2013), 1237-1251.

[9] R. Ponraj and S. Sathish Narayanan, "***Difference cordial labeling of some derived graphs",*** International journal of Mathematical combinatorics, **4**(2014), 37-48.

[10] R. Ponraj, S. Sathish Narayanan and R. Kala***, "A note on difference cordial graphs",*** Palestine Journal of Mathematics, **4(1)** (2015), 189″U197.

[11] R.Ponraj, S.Sathish Narayanan and R.Kala, "***Difference cordiality of product related graphs***, Tbilisi Mathematical Journal, **8(2)**(2015), 41-47.

[12] Rosa. A ***"On Certain Valuations of the vertices of a graph"***, Theory of graphs (Inter-nat. Symposium, Rome, July 1966), Gordon and Breach, N.Y. and Dunod Paris 349-355.

[13] M.A.Seoud and Shakir M. Salman, "***On difference cordial graphs***", Mathematica Aeterna, **5**, 2015, no. 1, 105- 124.